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Stability of Blocks Around Circular Openings

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ABSTRACT: For tunnels excavated through fractured rock, tunnel stability is governed, in part, by the stability of individual keyblocks formed by intersecting discontinuities, such as bedding planes or fractures. A variety of factors affects the stability of these keyblocks. These factors include block geometry, in-situ stresses, joint shear strength, tunnel depth, and tunnel diameter. Determining the support force that ensures stability of a particular keyblock becomes a statically indeterminate problem when the effects of in-situ stresses are considered. Several numerical models have been developed to handle this indeterminacy. However, these models tend to oversimplify the material properties of the surrounding rock mass. In this paper, a method is presented to determine keyblock stability around circular excavations independent of most material property assumptions. Using the optimization method of linear programming and the Kirsch solution for stresses around circular openings in elastic materials, boundaries on the support force required for keyblock stability are determined. An example is included which examines the various effects of the aforementioned rock and tunnel physical properties.

Introduction

In any underground excavation, tunnel stability is of utmost importance. For tunnels excavated through a rock mass, the presence of discontinuities, such as bedding planes or fractures, can create a potentially hazardous environment. When these discontinuities and the tunnel excavation intersect, a variety of rock blocks can be created, as shown in Figure 1a. In many strong igneous and metamorphic rocks, the stability of excavations at depths less than 500m below surface depends on the stability of these blocks and wedges (Hoek and Moy, 1993). For the example shown in Figure 1, failure of the primary keyblock directly above the tunnel can lead to progressive failure of the surrounding rock mass. However, as shown in Fig 2, if sufficient support force **Q** is applied to stabilize this keyblock, the remaining rock blocks, and consequently the tunnel, will remain stable as well.

The support force **Q** required for stability of keyblocks can be estimated by numerical techniques or analytical methods such as block theory (Goodman and Shi, 1985) or the relaxation method (Brady and Brown, 1993). Block theory is based on a kinematic analysis of removable keyblocks. Based on the frictional strength of the joints, the blocks are analyzed for kinematic stability under the action of gravity. However, block theory ignores the potentially stabilizing effect of in-situ stresses on keyblocks. Neglecting these tractions can lead to over-conservative, non-efficient designs. However, consideration of these in-situ stresses creates a statically indeterminate situation,

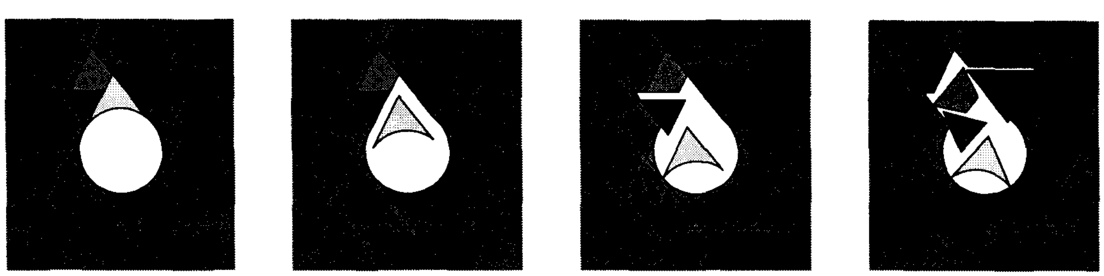

Figure 1 Keyblocks are formed around an excavation due to the presence of discontinuities. Failure of one keyblock can lead to a progressive failure of the rock mass

significantly increasing the difficulty of the keyblock stability problem.

A variety of numerical methods, including DDA (Shi, 1988) and UDEC (Cundall and Hart, 1993), have been applied to this problem. The pitfall of these models is that they are based on the constitutive relationships between the rock mass and discontinuities. Therefore they require material properties, such as Young's modulus and Poisson's ratio, to be either experimentally determined or assumed. Not only is it expensive and time consuming to determine these quantities, but these values are also often highly variable within a rock mass, whereas numerical models assume them to be constant throughout.

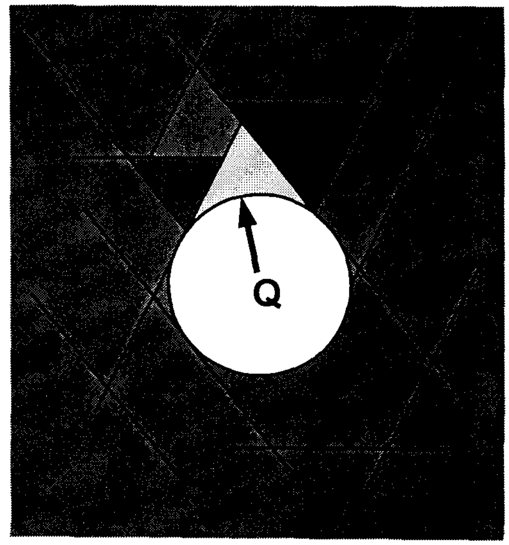

Figure 2 Application of a support force, **Q**, can stabilize the keyblock and surrounding rock mass.

Mauldon et al. (1999b) have previously presented a method based on limit analysis to determine the support force required for keyblock stability around excavations of rectangular cross-section. This method requires few assumptions to be made concerning the rock mass material properties, while still considering the effects of in-situ stresses. The analysis is based on static equilibrium of the block and the normal and shear strengths of the joints bounding the block. One drawback of the limit analysis method is that it requires an estimate of the initial normal and shear surface forces acting on the block face in order to compute limiting support forces for stable and unstable blocks.

In this paper, the limit approach is applied to determination of keyblock stability around circular excavations. Stability of rock around circular excavations has been discussed elsewhere (Kumar, 1997; Martin et al., 1997). Because block forces are statically indeterminate, estimation of the initial forces acting on the keyblock is difficult. The method discussed here considers the effects of in-situ stresses without requiring a value for the initial surface forces. Making use of the Kirsch solution for stresses around a hole in an elastic material, limiting values for stable and unstable keyblocks can be determined. An example is included to show the effects of block orientation, friction angle, and stress ratio on stability.

Problem

Determining the support force required for stability of keyblocks created by the intersection of rock mass discontinuities and excavations becomes a statically indeterminate problem when in-situ stresses are considered. Consider the cross-section shown in Figure 3 of a tunnel of radius r excavated at depth z. Two intersecting fractures, with static friction coefficients $\mu_1 = \tan\varphi_1$ and $\mu_2 = \tan\varphi_2$, create a keyblock of height h with apical angle β in the crown of the tunnel. The maximum keyblock occurs when joints 1 and 2 are tangent to the excavation surface as shown in Figure 3. We say the keyblock is oriented at an angle ψ, where ψ is

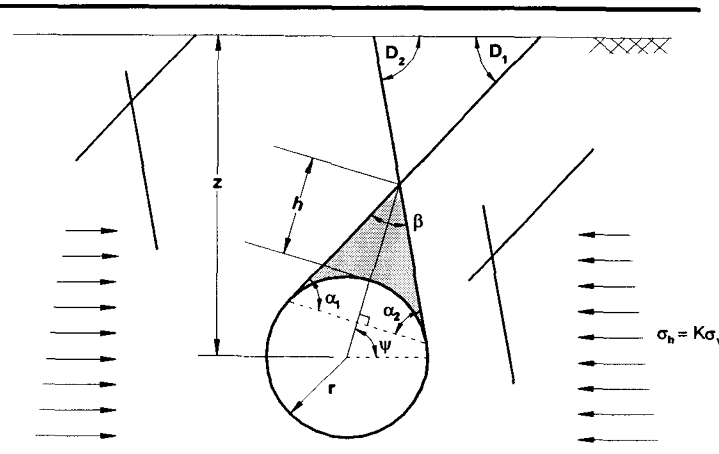

Figure 3 A tunnel of radius r at depth z. A keyblock, oriented at angle ψ, of height h is formed in the tunnel periphery

the angle from horizontal to a radial plane passing through the center of the tunnel and the keyblock apex (Fig3). Based on β, interior angles (α_1, α_2) are defined relative to the direction perpendicular to the radial plane. Here, only symmetric keyblocks with $\alpha_1 = \alpha_2 = \alpha$ are considered. Therefore, for a block of orientation ψ with apical angle β, $\alpha = (\beta - \pi)/2$. Consequently, the dip angles D_1 and D_2, are $D_1 = \alpha + \psi - 90$ and $D_2 = \alpha - \psi + 90$.

Horizontal and vertical in-situ stresses, σ_h and σ_v, respectively, occur within the rock mass. The horizontal stress can be represented as a proportion, K ($\sigma_h = K\sigma_v$), of the vertical stress to the horizontal stress ratio. From the in-situ stresses, normal and shear surface forces ($\mathbf{N_1}$, $\mathbf{N_2}$, $\mathbf{S_1}$, $\mathbf{S_2}$) act on the joints of the keyblock as shown in Fig 4a. However, these forces cannot be found from statics alone, as they depend partly on the mechanical response of the rock (Shi, 1988; Brady and Brown, 1993; Cundall and Hart, 1993; Mauldon et al 199a). Rather than estimating the surface forces, it is considerably easier and more reliable to estimate the normal stress, and the resultant normal force **F**, acting on the radial plane (Fig 4b).

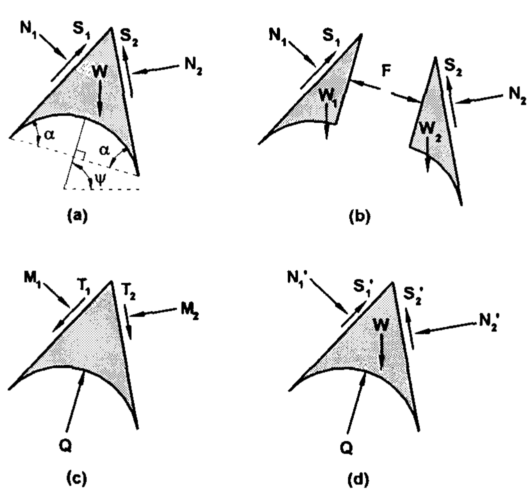

Figure 4 (a) Initial keyblock., (b) Cutting along a radial plane produces a normal force F, (c) Application of Q produces reactions on the joints, (d) Final keyblock

Application of the support force Q produces normal and shear reactions ($\mathbf{M_1}$, $\mathbf{M_2}$, $\mathbf{T_1}$, $\mathbf{T_2}$) on the joint surfaces (Fig. 4c). Again, because of the indeterminacy of the system, the magnitudes of these reactions cannot be immediately determined. By superposition, the final forces ($\mathbf{N_1'}$, $\mathbf{N_2'}$, $\mathbf{S_1'}$, $\mathbf{S_2'}$) on the keyblock, which are again indeterminate, can be found from the combination of the initial forces and the reactions to **Q**. Because the surface forces are statically indeterminate, a method other than statics must be used to determine the required force **Q** that will insure stability of the keyblock.

Based on equations of equilibrium and kinetic constraints for stable and unstable keyblocks, optimization techniques can be used to determine the limiting values of support for stable and unstable blocks. This idea of defining boundaries for stable and unstable blocks has been discussed elsewhere (Mauldon and Zhao, 1995; Mauldon et al., 1997a,b). Experience tells us that for a sufficiently large support force, the block will be stable, and conversely, for insufficient support, or perhaps negative forces, (i.e. loading forces), the block will be unstable. From optimization within the constraints of equilibrium and stability, it is possible to obtain two values: Qmin stable and Qmax unstable. Qmin stable is the smallest value Q for which the block could remain stable. Similarly, Qmax unstable is the largest value for which the block could remain unstable. Therefore, for any Q less than Qmin stable, the block will definitely be unstable. For any Q larger than Qmax unstable the block will be definitely stable.

For statically determinate systems, Qmin stable = Qmax unstable = Q for limiting equilibrium, as shown in Figure 5a. However, for the keyblock problem, there is a region between Qmin stable and Qmax unstable where the mechanical response of the keyblock cannot be determined. For any Q between Qmin stable and Qmax unstable, the keyblock will be potentially unstable (Fig 5b).

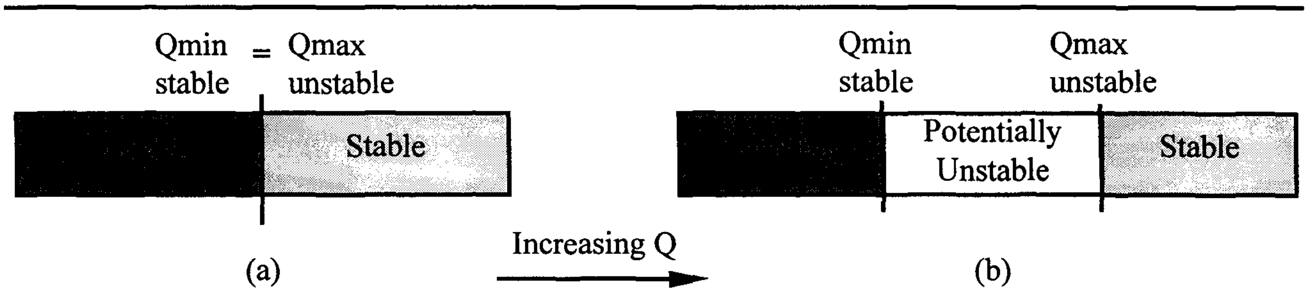

Figure 5 Regions of stable, unstable, and potentially unstable for (a) a determinate system and (b) an indeterminate system.

Block Forces

Initial Forces and the Kirsch Solution

The keyblock is subjected to a set of initial surface forces ($\mathbf{N_1}$, $\mathbf{N_2}$, $\mathbf{S_1}$, $\mathbf{S_2}$) and self-weight, $\mathbf{W}$. Although the exact magnitudes of these forces (N_1, N_2, S_1, S_2) can not be statically determined, it can be said that the forces must satisfy equilibrium. It should be noted that only force equilibrium is required. Because the exact location of these forces is unknown, moment equilibrium is not enforced. Force equilibrium for the block in Figure 4a is defined by the vector equation

$$\mathbf{N_1} + \mathbf{N_2} + \mathbf{S_1} + \mathbf{S_2} + \mathbf{W} = \mathbf{0}\,. \tag{1}$$

This equation can be separated into two scalar equilibrium equations, representing equilibrium in orthogonal directions in the plane of the tunnel cross-section. The sign convention used for the forces is that compression is taken positive for normal forces, and shear forces acting toward the apex of the keyblock are positive. From Figure 4a, parallel to the radial plane, we have:

$$(N_1 + N_2)\cos\alpha - (S_1 + S_2)\sin\alpha + W\sin\psi = 0\,. \tag{2}$$

From Figure 4b, perpendicular to the radial plane, equilibrium is defined by:

$$N_1\sin\alpha + S_1\cos\alpha + W_1\cos\psi = F\text{ , and} \tag{3a}$$

$$N_2\sin\alpha + S_2\cos\alpha - W_2\cos\psi = F\text{ , where} \tag{3b}$$

$$W_1 = W_2 = W/2\,. \tag{3c}$$

The magnitude F of the normal force must be estimated. There are multiple ways to determine a value for this force. One common field method is flat-jack testing. Flat-jack testing allows determination of the normal stresses acting on a radial plane. From these stresses the engineer can then estimate the force F on the radial plane. Another method to determine F is by utilizing the well-known Kirsch solution for stresses in an elastic material containing a hole. For circular excavations through large rock masses stressed below the elastic limit and containing widely spaced and tightly pre-compressed or healed joints, the Kirsch solution can be used to estimate the stresses acting around a circular excavation (Goodman 1989). Outside the tunnel of radius r, the Kirsch solution provides the normal stress, σ_ψ, at a point on a radial plane oriented at angle ψ. The normal force F on the radial plane can be found by integrating the Kirsch solution over the height of the keyblock from the excavation boundary to the block apex. After integrating, we find that

$$F = \sigma_v\left[\frac{K+1}{2}\left(h+r-\frac{r^2}{h+r}\right) + \frac{K-1}{2}\left(h+r-\frac{r^4}{(h+r)^3}\right)\cos 2\psi\right]. \tag{4}$$

This method of estimating F will be used throughout this paper.

Force reactions

Applying a support force of magnitude Q produces reactions which alter the normal and shear surface forces (Fig. 4c). Again, because of the indeterminacy of the problem, the magnitudes of the reactions are unknown. However, these reactions must also satisfy equilibrium as described by the vector equation:

$$\mathbf{M_1} + \mathbf{M_2} + \mathbf{T_1} + \mathbf{T_2} + \mathbf{Q} = \mathbf{0}. \tag{5}$$

This vector equation can be separated into two scalar equations. For equilibrium parallel to the radial plane, we have the following:

$$(M_1 + M_2)\cos\alpha + (T_1 + T_2)\sin\alpha = Q\,, \tag{6a}$$

and for equilibrium in the perpendicular direction, we have

$$(M_1 - M_2)\sin\alpha + (T_2 - T_1)\cos\alpha = 0\,. \tag{6b}$$

The magnitudes of the force reactions subscribe to the same sign convention as defined for the initial surface forces.

Final forces

The vector sum of the initial forces and the reactions to **Q**, give the final forces acting on the block (Fig 4d). Mauldon et al. (1999b) showed that for a support force acting in the direction of the block apex, the magnitudes of the aforementioned forces are related by the following scalar equations:

$$N_1' = N_1 + M_1 \tag{7a}$$

$$N_2' = N_2 + M_2 \tag{7b}$$

$$S_1' = S_1 - T_1 \tag{7c}$$

$$S_2' = S_2 - T_2\,. \tag{7d}$$

Again, the magnitudes of the final forces ascribe to the same sign convention as assumed for the initial forces and the force reactions.

Constraints

Stability and instability

The same definitions as used by Mauldon et al. (1999b) for stable and unstable blocks will be used with this model. Block failure is assumed to be associated with either shear or tensile failure of the joint planes. Allowing for potential rotational failure of obtuse keyblocks (Mauldon & Goodman 1996, Tonon 1998), definitions of stable and unstable keyblocks are developed for both acute and obtuse blocks. A block is either acute or

Table 1. Stable and Unstable definitions

Stable	Unstable
Acute	
$N_1 \geq 0$ -and- $N_1 \geq S_1/\mu_1$ -or- $N_2 \geq 0$ -and- $N_2 \geq S_2/\mu_2$	$N_1 \leq 0$ -or- $N_1 \leq S_1/\mu_1$ -and- $N_2 \leq 0$ -or- $N_2 \leq S_2/\mu_2$
Obtuse	
$N_1 \geq 0$ -and- $N_1 \geq S_1/\mu_1$ -and- $N_2 \geq 0$ -and- $N_2 \geq S_2/\mu_2$	$N_1 \leq 0$ -or- $N_1 \leq S_1/\mu_1$ -or- $N_2 \leq 0$ -or- $N_2 \leq S_2/\mu_2$

obtuse depending on whether the apical angle is less than or greater than $\pi/2$, respectively. Because failure in the acute case must be a result of translation and not rotation (Mauldon et. al, 1999b), failure of both joints is required. However, because rotational failure can occur in the obtuse case, shear or tensile failure of only one joint is required in the obtuse case. Therefore, we have the definitions for stable and unstable blocks as given in Table 1. The constraint of limiting **Q** to the middle third of the block for rotational failure in the obtuse case is also enforced as defined in Wu et. al (1999).

Valid and Invalid Forces

Not only are stable and unstable definitions of blocks defined, but shear and normal constraints can be placed on the initial and final surface forces as well. The forces must satisfy equilibrium as previously stated, and for forces to be "valid," or physically possible to achieve, forces must not exceed the strength limits of the joint surfaces. Initially, a set of final forces is considered physically valid if both the following are true (the joints are assumed to have zero cohesion):

$$\text{(a) } N_i \geq \frac{|S_i|}{\mu_i} \quad \text{and} \quad \text{(b) } N_i \geq 0. \qquad i = (1,2)\ . \qquad (8)$$

Similarly, a set of final forces is considered valid if and only if the following are true:

$$\text{(a) } N_i' \geq \frac{|S_i'|}{\mu_i} \quad \text{and} \quad \text{(b) } N_i' \geq 0 . \qquad i = (1,2) . \tag{9}$$

However, eqs. 8a and 9a imply 8b and 9b, respectively. Therefore, in the initial case, valid forces can be defined as

$$N_i \geq \frac{|S_i|}{\mu_i} \qquad i = (1,2) . \tag{10}$$

Likewise, in the final case, valid forces can be defined as

$$N_i' \geq \frac{|S_i'|}{\mu_i} \qquad i = (1,2) . \tag{11}$$

Optimization

As previously stated, the goal is to determine Qmin stable and Qmax unstable using optimization techniques. The premise of optimization is to find the optimal value (maximum or minimum) of some function, known as the objective function, within the constraints limiting the variables defining that objective function. The use of optimization techniques in a geotechnical environment has been discussed elsewhere (Chuang, 1992; Araujo et. al, 1996). Here, the idea is to optimize some function of Q to find the minimum Q required for stability and the maximum Q for possible instability. The optimization technique of linear programming was applied using Maple mathematical software (Char et al., 1991).

By setting eq. 6a to be the objective function, the remaining equations of equilibrium, superposition, stability and instability, and validity, linear programming can be used to perform the optimizations. Qmin stability is found by optimizing eq. 6a subject to the constraints of equilibrium, stability, and validity. Qmax instability is found by optimizing eq. 6a subject to the constraints of equilibrium, instability, and validity.

Example

To examine the effects of block orientation, friction coefficients, and stress ratio, a simple example is considered. A tunnel of radius 5m is excavated at a depth of 100m. A representative keyblock with $\beta = 60°$ is chosen. Optimizations are performed on identical blocks every 10° around the tunnel.

First, the effects of friction angle are examined. For this example, the horizontal stress ratio is 2, with friction coefficients ranging from 0.2 to 1.0 examined. The results, plotted in terms of the ratio Q/W, are shown in Figure 6. It should be noted that in Figure 6, the analysis for $\mu = 0.6$ has been omitted. As an illustration of the results of Figure 6, consider the stability of a $\beta = 60°$ keyblock in the crown of a tunnel ($\psi = 90°$) with $\mu =$

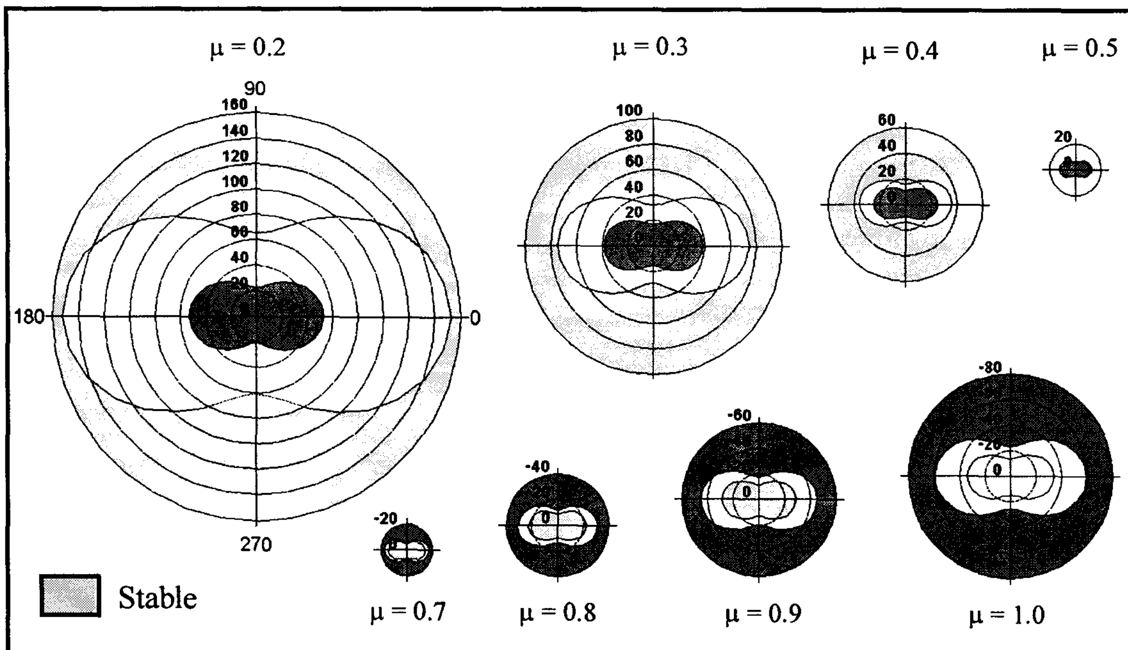

Figure 6 Keyblocks with $\beta = 60°$ are analyzed under varying friction coefficients with $k = 2$. Results are plotted in terms of Q/W.

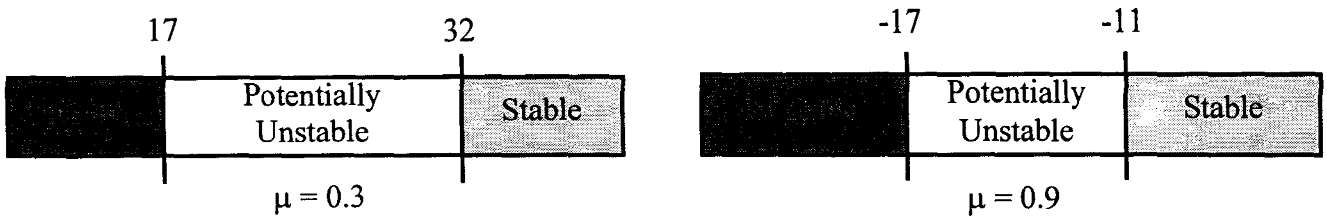

Figure 7 For keyblocks with $\mu = 0.3$ and $\mu = 0.9$ in the crown of the tunnel, regions for stable, unstable, and potentially unstable, based on the results from Figure 6.

0.3. From Figure 6 for $\mu = 0.3$, we find that for Q/W greater than 32 the block will be stable, for Q/W less than 17 the block will be unstable, and for support between 17 and 32 times the weight of the keyblock, the stability is unknown, as also shown in Figure 7. From these results we know that under the given conditions, the block will be unstable without any support (Q/W = 0). For $\mu = 0.9$, we see from Figure 6 that at Q/W = 0, the block is stable. These results are also shown in Figure 7, where we see that for any applied load less than 11 times the weight of the block, the keyblock will remain stable.

For small friction coefficients, all blocks, independent of orientation, are unstable under their own and weight and require additional support. However, as the friction coefficient increases, the blocks, regardless of position, become increasingly stable. Also, as blocks become either increasingly stable or unstable, the region of potentially unstable, corresponding to the uncertainty in the mechanical response of keyblocks also increases.

Next, the effects of stress ratio are examined with the friction coefficient fixed at 1.0. Values of k ranging from 0.5 to 3.0 are analyzed. The results are shown in Figure 8. Regardless of stress ratio, all blocks are stable without additional support. As the stress ratio increases to 1.0, the requirements for stable and unstable blocks become uniform around the tunnel (as shown by the circular regions). As the stress ratio increases, blocks

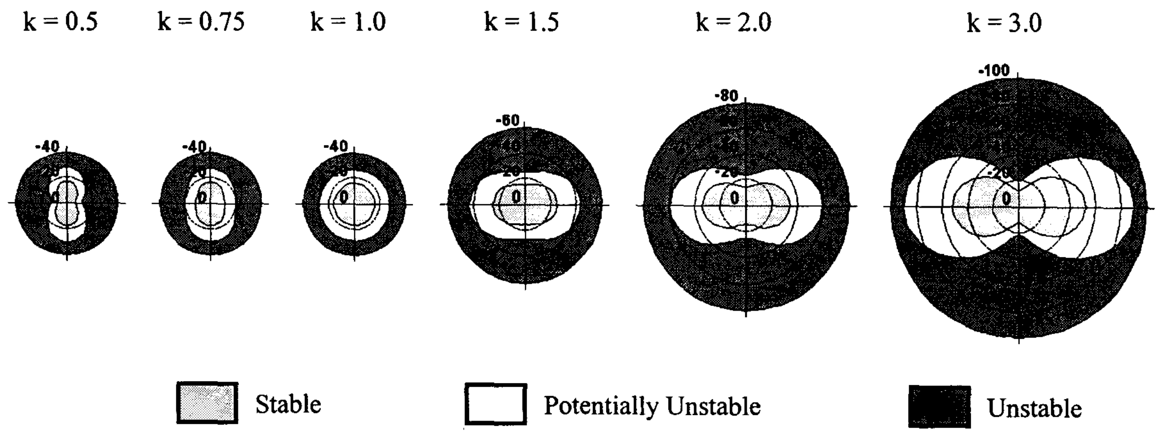

Figure 8 Keyblocks with $\beta = 60°$ are analyzed under varying stress ratios with $\mu = 1$. Results are plotted in terms of Q/W.

in the roof and floor become less stable, while blocks in the sidewalls increase in stability.

Conclusions

Rock blocks created by discontinuities can pose serious problems to the geotechnical engineer. Numerous methods are available to assist the rock engineer in design of support to ensure stability of the keyblocks. While commonly used in practice, many of these methods tend to oversimplify the keyblock stability problem. Optimization techniques can be used to determine the bounds on support force required for keyblock stability. As shown in the example, two factors, which play a primary role in the stability of keyblocks around circular excavations are friction coefficient and stress ratio. Other factors that influence keyblock stability are block geometry, tunnel depth, and tunnel size.

Acknowledgement

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